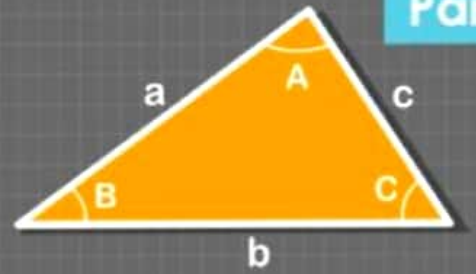
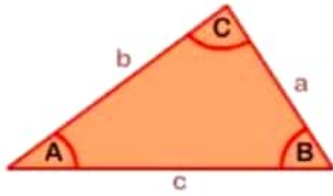


SOLUTION OF TRIANGLE



LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

PROJECTION FORMULA

$$a = b \cos C + c \cos B$$

$$b = c \cos A + a \cos C$$

$$c = a \cos B + b \cos A$$

AREA OF TRIANGLE

$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} = \frac{1}{2} bc \sin A$$

$$\text{Area} = \frac{1}{2} ca \sin B$$

NAIPER'S ANALOGY

$$\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$$

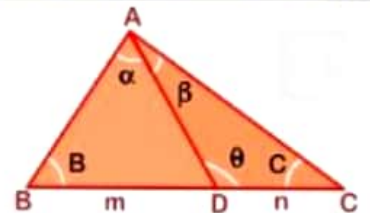
$$\tan \frac{C-A}{2} = \frac{c-a}{c+a} \cot \frac{B}{2}$$

$$\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$$

M-N THEOREM

$$(m+n) \cot \theta = m \cot \alpha - n \cot \beta$$

$$(m+n) \cot \theta = n \cot B - m \cot C$$



TRIGONOMETRIC HALF ANGLES

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$$

$$\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$\text{where, } s = \frac{a+b+c}{2}$$

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$$

$$\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$$

$$\text{where, } s = \frac{a+b+c}{2}$$

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$$

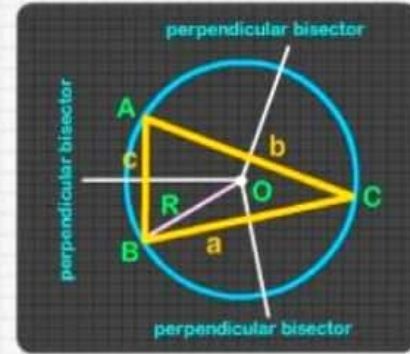
$$\tan \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$$

$$\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

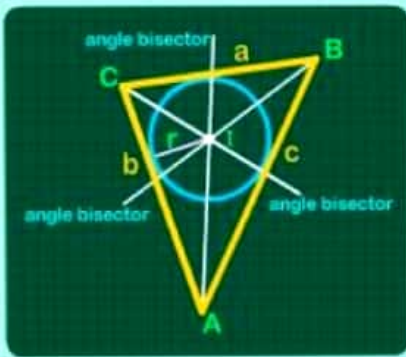
$$\text{where, } s = \frac{a+b+c}{2}$$

$R \rightarrow$ Circumradius of ΔABC .

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R; \Delta = \frac{abc}{4R}$$



$$a \cos B \cos C + b \cos C \cos A + c \cos A \cos B = \frac{\Delta}{R}$$



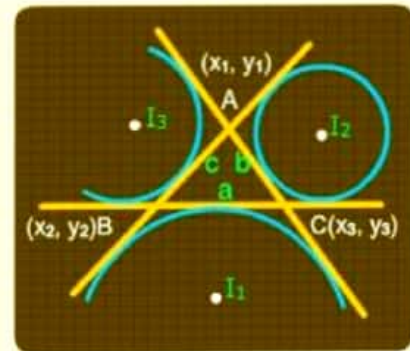
$r \rightarrow$ Inradius of ΔABC .

$$r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}} = \frac{b \sin \frac{C}{2} \sin \frac{A}{2}}{\cos \frac{B}{2}} = \frac{c \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{C}{2}}$$

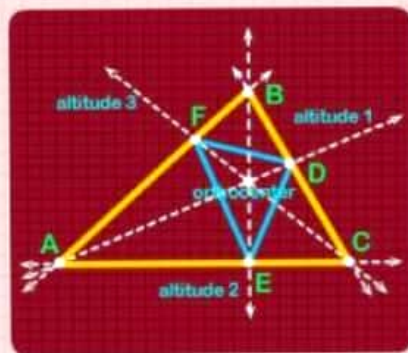
$$r = (s - a) \tan \frac{A}{2} = (s - b) \tan \frac{B}{2} = (s - c) \tan \frac{C}{2}; r = \frac{\Delta}{s}; s = \frac{a + b + c}{2}$$

$r_1, r_2, r_3 \rightarrow$ are radii of excircles of ΔABC .

$$r_1 = \frac{\Delta}{s - a}; r_2 = \frac{\Delta}{s - b}; r_3 = \frac{\Delta}{s - c}; s = \frac{a + b + c}{2}$$



$$r_1 = s \tan \frac{A}{2}; r_2 = s \tan \frac{B}{2}; r_3 = s \tan \frac{C}{2}$$



ORTHOCENTRE AND PEDAL TRIANGLE

ORTHOCENTRE - Point of intersection of 3 altitudes.

PEDAL - Pedal triangle is formed by joining the feet of altitudes.

Area of $\Delta DEF = 4 \Delta \cos A \cos B \cos C$

where Δ is the area of triangle ABC.

LENGTH OF ANGLE BISECTOR MEDIAN & ALTITUDE

Length of an angle bisector from the angle A = $\beta_a = \frac{2bc \cos \frac{A}{2}}{b + c}$

Length of median from the angle A = $m_a = \frac{1}{2} \sqrt{2b^2 + 2c^2 - a^2}$

Length of altitude from the angle A = $A_a = \frac{2\Delta}{a}$ $\Delta \rightarrow$ Area of triangle ABC.

